Nonlinear Static (Pushover) Analysis

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Topics

- What is nonlinear static analysis?
- How is it done?
- What are the drawbacks to its use?
- When can (should) it be used?
What is nonlinear static analysis

- One of four analysis procedures embodied in FEMA 356 / ASCE 41 and commonly used in performance-based design approaches
  - Linear static
  - Linear dynamic
  - Nonlinear static (pushover)
  - Nonlinear dynamic

Advantages

- Intuitive and visual method
  - Allows engineers to “understand” structure’s nonlinear behavior and progression of damage with increasing ground motion intensity
- Simple to perform (with or without nonlinear analysis software)
- Does not require selection and scaling of ground motions
Disadvantages

- Static
  - Can not directly account for dynamic structural behavior

Nevertheless, nonlinear static analysis can be effectively used for performance assessment and design of many types of structures.

How is it done

- Basic Steps
  - Model structure (and important nonstructural components)
  - Perform a series of linear analyses on sequentially degraded models as damage is predicted
  - Develop pushover curve
  - Determine effective dynamic properties
  - Determine demand lateral displacement for design ground motion
  - Check adequacy of elements for force and deformation demands at design lateral displacement
Getting Started

- Understand the structure before attempting a nonlinear analysis
  Identify:
  - critical elements
  - probable yield/failure modes
  - importance of foundation to building response
  - importance of torsional behavior and need for 3D modeling

Identify Critical Elements

- Nonlinear analysis is complex and it is desirable to minimize the number of elements modeled
- Sufficient elements must be included to capture important behavior modes
- Critical elements:
  - all members of lateral force resisting system
  - any members sensitive to large lateral displacements
  - any elements that affect the above
Typical Critical Elements

- Steel frame structures
  - moment connected frames
    - Beams, columns, panel zones
  - braces and link beams
  - masonry & concrete infills
  - foundations

- Concrete structures
  - girders - column frames
  - flat slab - column frames
  - shear walls
  - architectural walls, spandrels & parapets cast integrally with structure
  - foundations

Identify Probable Yield/Failure Modes

- Structures can exhibit a variety of inelastic behaviors
- It is critically important to model those that are likely to occur in the structure analyzed

Joint Shear
Column Shear
Wall Shear
Wall Flexure
Foundation rocking
Identify Importance of Torsional Response

- Two dimensional analyses can be used for torsionally regular buildings
- Three dimensional analyses are required for irregular buildings

Modeling Element Properties & Strengths

- Best estimates (expected values) rather than design or specified strengths should be used in determining material capacities
  - use of artificially low nominal strengths will result in under estimates of strength demands on some elements
    - Expected Steel yield - 1.25Fy
    - Expected concrete compressive strength - 1.33f'c
Modeling Steel Moment Frames

Before building the pushover model, determine where the yielding will occur:
- Girder
  - column face
  - interior span
- Column
- Panel Zone

Modeling Steel Braced Frames

- Tension Braces

\[ K = \frac{EA}{L} \]
\[ \Delta_y = \frac{F_y L}{E} \]
\[ \Delta_u = 5\Delta_y \]

- Compression Braces

\[ K = \frac{EA}{L} \]
\[ P_c = 1.7F_uA \]
\[ \Delta_c = 1.7\frac{F_u L}{E} \]
\[ \Delta_u = 3\Delta_c \]
Modeling Concrete Structures

- Before proceeding - determine critical behavior mode for each element:
  - flexure
  - shear
  - end connection

- Except for low-tensile demand elements, use “effective” cracked section properties

Modeling Concrete Structures

**Flexure**

- Cracked section properties-\(E_{I,cr}\):
  - Beams \(0.5E_I\_g\)
  - Prestressed Beams \(E_I\_g\)
  - Columns (no net tension) \(0.7E_I\_g\)
  - Columns (net tension) \(0.5E_I\_g\)
  - Walls uncracked \(0.8E_I\_g\)
  - Walls cracked \(0.5E_I\_g\)

Use of M-\(\phi\) relations obtained from BIAx and straight mechanics of materials approaches will underestimate stiffness.
Modeling Concrete Structures
Flexure

- Use elastic-plastic model
- Ultimate deformation determined based on concrete strain
  - $\varepsilon_u$ dependent on
    - confinement provided
    - lateral support for longitudinal steel

\[ \varepsilon_u = 0.005 + 0.1 \rho_c \left( \frac{f_i}{f_c} \right) \leq 0.02 \]

or:

- $\varepsilon_u = 0.005$ for $\frac{s}{d_b} \geq 16$
- $\varepsilon_u = 0.02$ for $\frac{s}{d_b} \leq 8$

\[ \theta_y = \phi_y l_p \quad \theta_u = \phi_u l_p \]

\[ l_p = \frac{d}{2} \]

\[ \phi_u = \varepsilon_u c \]

Neutral axis depth “c” can be evaluated at strain of 0.003
1. Select a loading pattern for the structure
   - Loading pattern should produce a deflected shape in the structure similar to that it would undergo in earthquake response

- Loading Pattern Alternatives
  - Inverse triangular
  - Rectangular
  - First mode
  - Modal dynamic
  - Modal dynamic variant
  - Multi-mode
Inverse triangular

- Approximates first mode response shape of regular structures
- Forces distributed with height by:

\[ F_x = \frac{w_x h_x V}{\sum w_i h_i} \]

Rectangular

- Approximates first mode response of structure with very soft stories, or post-yield response of structures with weak first stories

\[ F_x = \frac{W_x V}{\sum W_i} \]
First Mode

- First mode response obtained from a modal analysis

\[ F_x = \frac{w_x \Delta_x}{\sum \Delta_i h_i} V \]

Modal dynamic

- Deformed shape of combined modes

\[ F_x = \frac{w_x \Delta_x}{\sum \Delta_i h_i} V \]
Loading Pattern

- FEMA 356 requires use of at least:
  - Inverse triangular or first mode
  - Rectangular
- FEMA 440 found that there is not substantial difference in the accuracy produced by the various load patterns

Develop Pushover Curve

[Graph showing displacement and applied shear]
**Determine Effective Dynamic Properties**

- Initially, perform elastic modal analysis to determine fundamental period of structure, $T$
- Determine initial stiffness, $k_i$ from pushover curve as $V_1/\Delta_1$
- Determine effective stiffness, $k_e$ at 60% of yield force from pushover curve

**Determine Initial Stiffness**

![Diagram showing determination of initial stiffness](image_url)
Determine Effective Stiffness

- Applied Shear $V$
- Displacement at Reference Point
- Judgmentally selected yield force level $V_{y1}$

Determine Effective Period

$$T_e = \sqrt{\frac{k_e}{k_i}} T$$
Determine Demand Lateral Displacement

- FEMA 356 permits two alternative methods
  - Coefficient (displacement modification) method
  - Capacity spectrum (equivalent linearization) method
    - As presented both are significantly flawed
    - FEMA 440 suggests important improvements to both methods
- Only coefficient method as modified by FEMA 440 presented here

\[
\Delta_T = C_0 C_1 C_2 \frac{T_e^2}{4\pi^2} S_a(T_e) g
\]

- \( C_0 \) = modal shape factor obtained from modal analysis
- \( C_1 \) = modifier to account for difference between elastic and elastic-perfectly plastic response
- \( C_2 \) = modifier to account for effects of hysteretic pinching (stiffness degradation)
- \( S_a \) = spectral acceleration from design spectrum at effective period of structure
- \( g = 386.4 \text{ in/sec}^2 \)
Coefficient $C_1$

$$C_1 = 1 + \frac{R - 1}{a T_e^2}$$

For periods of 1 second or greater, $C_1$ may be taken as having a value of unity.

$$R = \frac{V_{y-j}}{\alpha S_a(T_e)W}$$

$\alpha$ = modal mass participation factor

$a$ = coefficient dependent on site class:
- 130 for site class B
- 90 for site class C
- 60 for site class D

For periods of 1 second or greater, $C_1$ may be taken as having a value of unity.
Coefficient $C_2$

$$C_2 = 1 + \frac{1}{800} \left( \frac{R - 1}{T} \right)^2$$

For periods greater than 0.7 seconds, $C_2$ may be taken as unity.

From FEMA 440

Check element adequacy at $\Delta_t$

- Once the target displacement for a given spectrum is determined, each element must be evaluated at the demand displacement to assure that neither its strength demand or deformation demand exceeds permissible capacities for the desired performance level.
- Capacities can be obtained from FEMA 356 hysteretic tables, or directly off laboratory test data.
What are limitations of nonlinear static analysis

- An approximate procedure
- Can not directly account for:
  - Cyclic behavior
  - Certain types of strength degradation
  - Dynamic collapse
  - Behavior of multiple degree of freedom structures
  - Randomness of response related to ground motion characteristics

Randomness of response

8 motions, all Class D sites, large strike slip events
Scaled to same spectral acceleration at 1.1 second
When is pushover analysis appropriate

- Generally, structures with initial periods of 1 second or less.
  - Only use if significant nonlinearity expected
  - If torsion is significant, pushover analysis may give an overly pessimistic understanding of structural behavior