Design of Concrete Moment Frames

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Topics to be Covered

• Preferred Lateral Mechanisms
• Code Provisions for Member Design
• Shear Design of Beams
• Column Confinement Design
• Joint Design
Structural Behavior of Frames

<table>
<thead>
<tr>
<th>Story Mechanism</th>
<th>Sway Mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undesirable</td>
<td>Desirable</td>
</tr>
</tbody>
</table>

Story Mechanism
Chapter 21 deals with members resisting earthquake loads

Performance Objectives of ACI 318

- **Strong column**
  - Avoid story mechanism

- **Hinge development**
  - Confined concrete core
  - Prevent rebar buckling
  - Prevent shear failure

- **Member shear strength**
- **Joint shear strength**
- **Rebar development**
Code Provisions

Beam Design

Factored axial compressive force $P_x \leq A_f f'/10$

Clear span $\ell_x \leq 4 \times$ effective depth

Width-to-depth ratio $b_x / h \geq 0.3$

Width $b_x \geq 10$ in.

Width $b_x \leq$ width of supporting member (measured on a plane perpendicular to the longitudinal axis of the flexural member) + distances on each side of the supporting member not exceeding three-fourths of the depth of the flexural member.

---

Code Provisions

Beam Design

[Diagram of beam design with dimensions and notes]

- Column $c_1 \times c_2$ (typ.)
- $\ell_x \geq 4d$
- $h$
- $d$
- $b_x$
- Larger of $0.3h$ or $10''$
- $\leq c_2 + 1.5h$

Section A-A
Code Provisions

Beam Design-Reinforcement

Minimum reinforcement $A_{rein}$ shall not be less than
\[ \frac{2f'_{c}d_{e}}{f_{y}} \] and \[ \frac{200f'_{c}d_{e}}{f_{y}} \]
at any section, top and bottom, unless provisions of 10.5.3 are satisfied.

The reinforcement ratio $\rho$ shall not exceed 0.025.

At least two bars must be provided continuously at both top and bottom of section.

Positive moment strength at joint face $\geq \frac{1}{2}$ negative moment strength provided at that face of the joint.

Neither the negative nor the positive moment strength at any section along the member length shall be less than $\frac{1}{4}$ the maximum moment strength provided at the face of either joint.

Code Provisions

Beam Design

\[ \rho_{min} = \frac{2\sqrt{f'_{c}f_{y}}}{200f_{y}}, \text{unless 10.5.3 is satisfied} \]

\[ \rho_{max} = 0.025 \]

Min. 2 bars continuous

$M_{n,e} \geq M_{n,e}/2$

$M_{n,f} \geq M_{n,f}/2$

$M_{n} \text{ or } M_{f} \geq (\text{max. } M_{n} \text{ at either joint})/4$

Note: transverse reinforcement not shown for clarity
## Code Provisions

### Beam Design-Splice Requirements

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.3.2.3</td>
<td></td>
</tr>
<tr>
<td>21.3.2.4</td>
<td></td>
</tr>
</tbody>
</table>

Lap splices of flexural reinforcement are permitted only if hoop or spiral reinforcement is provided over the lap length. Hoop and spiral reinforcement spacing shall not exceed:

- $d/4$
- 4 in.

Lap splices are not to be used:

- Within joints.
- Within a distance of $2h$ from the face of the joint.
- At locations where analysis indicates flexural yielding caused by inelastic lateral displacements of the frame.

Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7.

---

Lap splice confined and located outside potential hinge area

Hoop or spiral reinforcement

$s \leq d/4$

$s \leq 4$
## Code Provisions

### Beam Design-Lateral Reinforcement

<table>
<thead>
<tr>
<th>Hoops are required in the following regions of frame members:</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Over a length equal to 2h from the face of the supporting member toward midspan at both ends of the flexural member.</td>
<td>21.3.3.1</td>
<td></td>
</tr>
<tr>
<td>• Over lengths equal to 2h on both sides of a section where flexural yielding may occur in connection with inelastic lateral displacements of the frame.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where hoops are required, the spacing shall not exceed:

| • d/4 | 21.3.3.2  | 4 |
| • 8 x diameter of smallest longitudinal bar |          |   |
| • 24 x diameter of hoop bars |          |   |
| • 12 in. |          |   |

The first hoop shall be located no more than 2 in. from the face of the supporting member.

<table>
<thead>
<tr>
<th>Where hoops are required, longitudinal bars on the perimeter shall have lateral support conforming to 7.10.5.3.</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.3.3.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than d/2 throughout the length of the member.

<table>
<thead>
<tr>
<th>Stirrups or ties required to resist shear shall be hoops over lengths of members in 21.3.3, 21.4.4, and 21.5.2.</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.3.3.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Code Provisions

### Beam Design-Lateral Reinforcement

Hoops in flexural members shall be permitted to be made up of 2 pieces of reinforcement: a stirrup having seismic hooks at both ends and closed by a crosstie. Consecutive crossties engaging the same longitudinal bar shall have their 90-degree hooks at opposite sides of the flexural member. If the longitudinal bars secured by the crossties are confined by a slab on only one side of the flexural frame member, the 90-degree hooks of the crossties shall be placed on that side.

<table>
<thead>
<tr>
<th>Hoops in flexural members shall be permitted to be made up of 2 pieces of reinforcement: a stirrup having seismic hooks at both ends and closed by a crosstie.</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.3.3.6</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

Transverse reinforcement must also be proportioned to resist the design shear forces.

<table>
<thead>
<tr>
<th>Transverse reinforcement must also be proportioned to resist the design shear forces.</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.3.4</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>
Code Provisions

Figure 5 - Shop Reinforcement

Code Provisions
Code Provisions

Column Design

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4.1</td>
<td></td>
<td>Factored axial compressive force $&gt; A_f f_c / 10$</td>
</tr>
<tr>
<td>21.4.1.1</td>
<td>6</td>
<td>Shortest cross-sectional dimension measured on a straight line passing through the geometric centroid $\geq 12$ in.</td>
</tr>
<tr>
<td>21.4.1.2</td>
<td></td>
<td>Ratio of the shortest cross-sectional dimension to the perpendicular dimension $\geq 0.4$</td>
</tr>
</tbody>
</table>

Note:

c₁ is the dimension in the direction of framing and c₂ is the perpendicular dimension.
Code Provisions

Column Flexural Strength Requirements

The flexural strengths of columns shall satisfy the following:

\[ \sum M_{nc} \geq (6/5) \sum M_{ref} \]  \hspace{1cm} (21-1)

where \( \sum M_{nc} \) = sum of nominal flexural strengths of columns framing into the joint, evaluated at the faces of the joint. Column flexural strength shall be calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.

\( \sum M_{ref} \) = sum of nominal flexural strengths of the beams framing into the joint, evaluated at the faces of the joint.

In T-beam construction, where the slab is in tension under moments at the face of the joint, slab reinforcement within an effective slab width defined in 8.10 shall be assumed to contribute to \( M_{nc} \) if the slab reinforcement is developed at the critical section for flexure.

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4.2.2</td>
<td>—</td>
</tr>
</tbody>
</table>

Code Provisions

Column Flexural Strength Requirements

If Eq. (21-1) is not satisfied, the lateral strength and stiffness of the columns shall not be considered when determining the strength and stiffness of the structure, and the columns shall conform to 21.11. Also, the columns must have transverse reinforcement over their full height as specified in 21.4.4.1 through 21.4.4.3.

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4.2.1</td>
<td>—</td>
</tr>
<tr>
<td>21.4.2.3</td>
<td>—</td>
</tr>
</tbody>
</table>
## Code Provisions

### Column Longitudinal Reinforcement Design

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.4.3.1</td>
<td></td>
<td>The area of longitudinal reinforcement $A_w$ shall not be less than $0.01A_y$ and shall not exceed $0.06A_y$.</td>
</tr>
<tr>
<td>21.4.3.2</td>
<td>7</td>
<td>Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. Lap splices are permitted only within the center half of the member length, must be tension lap splices, and shall be enclosed within transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3.</td>
</tr>
</tbody>
</table>

### Figure 7 - Longitudinal Reinforcement Requirements

- Tension lap splice within center half of member length enclosed with transverse reinforcement per 21.4.4.2 and 21.4.4.3.
- $0.01A_y \leq A_w \leq 0.06A_y$

Section A-A
**Code Provisions**

<table>
<thead>
<tr>
<th>Code Provisions</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
</table>
| The transverse reinforcement requirements discussed in the following seven items need only be provided over a length \( L_e \) from each joint face and on both sides of any section where flexural yielding is likely to occur. The length \( L_e \) shall not be less than the largest of:  
  - Depth of member at joint face or at section where flexural yielding is likely to occur  
  - Clear span/B  
  - 18 in. | 21.4.4.4 | 8        |
| **Column Confinement Requirements**                                             |           |          |
| Ratio of spiral or circular hoop reinforcement \( \rho_s \) shall not be less than that given by:  
  \[
  \rho_s = 0.12 \frac{f_y}{f_{yf}} \geq 0.45 \left( \frac{A_{ch}}{A_{th}} - 1 \right) \left( \frac{f_y}{f_{yf}} \right) \quad (21-2) \text{ and } (10-6)  
  \] | 21.4.4.1(a) | 9        |

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**Code Provisions**

<table>
<thead>
<tr>
<th>Code Provisions</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
</table>
| Total cross-sectional area of rectangular hoop reinforcement for confinement \( A_{ch} \) shall not be less than that given by the following two equations:  
  \[
  A_{ch} = 0.3 \left( \frac{ab}{f_y} \right) \left[ \frac{A_{th}}{A_{ch}} - 1 \right] \quad (21-3)  
  \]  
  \[
  A_{ch} = 0.9 \left( \frac{ab}{f_y} \right) \quad (21-4)  
  \] | 21.4.4.1(b) | 8        |
| **Column Confinement Requirements**                                             |           |          |
| Transverse reinforcement shall be provided by either single or overlapping hoops. Crosses of the same bar size and spacing as the hoops are permitted, with each end of the crosspiece engaging a peripheral longitudinal reinforcing bar. Consecutive crosses shall be alternated end to end along the longitudinal reinforcement. | 21.4.4.1(c) |          |
| Eqs. (21-9) and (10-5) need not be satisfied if the design strength of the member core satisfies the requirement of the design loading combinations, including \( E' \), load effects for the earthquake effect. | 21.4.4.1(d) |          |
| If the thickness of the concrete outside of the confining transverse reinforcement > 4 in., additional transverse reinforcement shall be provided at a spacing ≤ 12 in. Concrete cover on the additional reinforcement ≤ 4 in. | 21.4.4.1(e) | 8        |
Code Provisions

**Columns supporting reactions from discontinued stiff members, such as walls, shall have transverse reinforcement as specified in 21.4.4.1–21.4.4.3 over their full height beneath the level at which the discontinuity occurs, if the factored axial compressive force related to earthquake effects > $A_f f_e / 10$. This transverse reinforcement shall extend into the discontinued member for at least the development length in tension $l_d$ of the largest longitudinal reinforcement in the column in accordance with 21.5.4.**

- If the lower end of the column terminates on a wall, transverse reinforcement per 21.4.4.1–21.4.4.3 shall extend into the wall for at least the development length $l_d$ of the largest longitudinal bar in the column at the point of termination.

- If the column terminates on a footing or mat, transverse reinforcement per 21.4.4.1–21.4.4.3 shall extend at least 12 in. into the footing or mat.

21.4.4.5  10

### Code Provisions

<table>
<thead>
<tr>
<th>Spacing of transverse reinforcement shall not exceed the smallest of:</th>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum member dimension/4</td>
<td>21.4.4.2</td>
<td></td>
</tr>
<tr>
<td>$6 \times$ longitudinal bar diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_o$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>where $4$ in. $\leq s_o = 4 + \frac{14 - h_b}{3} \leq 6$ in.</td>
<td>(21-5)</td>
<td>21.4.4.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Horizontal spacing of cross ties or legs of overlapping hoops $h_b$ shall not exceed 14 in. on center. Vertical bars shall not be farther than 6 in. clear from a laterally supported bar.</th>
<th>Sect. No.</th>
<th>21.4.4.3</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Where transverse reinforcement as required in 21.4.4.1–21.4.4.3 is not provided throughout the full length of the column, the remainder of the column shall contain spiral or hoop reinforcement with center-to-center spacing $s$ not to exceed:</th>
<th>Sect. No.</th>
<th>21.4.4.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>- $6 \times$ longitudinal bar diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 6 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Transverse reinforcement must also be proportioned to resist the design shear forces. 21.4.5
Code Provisions

Column Confinement Requirements

Provide additional transverse reinforcement if thickness > 4"

Alternate 90-deg hooks

$\delta_d \geq 3"$

$6d_d \geq 3"$

$5.6d_d \text{ extension}$

$A_{\text{bar}} \geq 0.3\beta_{b2} \left( A_{s} / A_{bc} \right)^{-1} f'_c / f_y$

$\frac{0.65b_d f'_c}{f_y}$

$h_x \leq 14" \text{ on center}$

$h_x = \max \text{ value of } h_x \text{ on all column faces}$

$4" \leq s_0 = 4 + (14 - h_x) / 3 \leq 8"$

Section A-A

*Provisions of 21.4.5 must also be satisfied*
Code Provisions

Column Spiral Hoop Requirements

Min. 6 longitudinal bars

Min. 3/8" diameter spiral rein.

Section A-A

Clear space

\[
\begin{align*}
&\text{clear space} \\
&\geq 1" \\
&\geq 1.33 \times \text{max. aggregate size}
\end{align*}
\]

\[
D_h \geq \begin{cases} 
0.12E / f_c & \\
0.45 \left( A_g / A_m \right) - 1 & \\
1.5 / f_c & 
\end{cases}
\]

Min. 6 longitudinal bars

Code Provisions

Column Supporting Discontinuous Stiff Members

Reinforcement not shown for clarity

Development length of largest longitudinal column reinforcement in accordance with 21.5.4

Transverse reinforcement per 21.4.4.1 - 21.4.4.3 over full height of columns

Development length of largest longitudinal column reinforcement in accordance with 21.5.4

Shearwall

Wall

Footing or mat

\geq 12"
Code Provisions

Undesirable
Desirable

Code Provisions

Hysteretic Behavior of Well Confined Concrete Columns
Moment Frame Example

Column Lines 2 and 7

Column Lines 3 to 6
Moment Frame Example

Reinforcement Layout

Shear Design of Beams

\[ V_e = \frac{M_{pr1} + M_{pr2}}{l_n} \pm \frac{w_u \ell_n}{2} \geq V_c \text{ by analysis} \]

If earthquake-induced shear force \( \frac{1}{2} V_e \) and \( P_u < \frac{A_g f_y}{20} \) then \( V_c = 0 \)

with

\[ f_s = 1.25 f_y, \quad \phi = 1.0 \]
Shear Design of Beams-Example

Max negative $M_u = 5834$ in-kips

$b = 22.5''$  $d = 29.6''$  $f'_c = 4$ ksi  $f_y = 60$ ksi

$A_{s\text{req'd}} = \frac{M_u}{\phi f_y (0.875d)} = \frac{5834}{0.9 \cdot 60 \cdot 0.875 \cdot 29.6} = 4.17 \text{ in}^2$

Choose: 2 #9 and 3 #8  $A_s = 4.37 \text{ in}^2$

$\rho = 0.0066 < 0.025$  OK

$\phi M_n = 6580$ in-kips  OK
Shear Design of Beams-Example

Positive $M_u$ at face of column = 4222 in-kips
(greater than $\frac{1}{2}(5834) = 2917$)

$b$ for negative moment is the sum of the beam width (22.5 in.) plus $1/12$ the span length (20 ft x 12 in./ft)/12,
b = 42.5 in.

\[
A_s \text{req'd} = \frac{M_u}{\phi f_y (0.9d)} = \frac{4222}{0.9} \cdot \frac{0.9}{60 \cdot 0.9 \cdot 29.6} = 2.94\text{in}^2
\]

Shear Design of Beams-Example

Choose 2 #7 and 3 #8  $A_s = 3.57$ in$^2$
\(\phi M_n = 5564 \text{ in-kips} \quad \text{OK}\)

Run 3 #8s continuous top and bottom
\(\phi M_n = 3669 \text{ in-kips}\)
This moment is greater than:
25% of max negative $M_n = 1459 \text{ in-kips}$
Max required $M_u = 834 \text{ in-kips}$
Shear Design of Beams-Example

Beam Reinforcement Layout

![Beam Reinforcement Diagram]

Total 6-#8

Shear Design of Beams-Example

\[ V_e = \frac{M_{pr1} + M_{pr2}}{\ell_n} + \frac{w_u \ell_n}{2} \geq V_e \] by analysis

If earthquake-induced shear force \( > \frac{1}{2} V_e \)

and \( P_u < \frac{A_d f_y}{20} \)

then \( V_e = 0 \)

with

\[ f_s - 1.25f_y, \quad \phi = 1.0 \]
Shear Design of Beams - Example

- Check maximum spacing of hoops within plastic hinge length (2d)
  - $d/4 = 7.4$ in.
  - $8d_b = 7.0$ in.
  - $24d_h = 9.0$ in.

Special Moment Resisting Frame Joints
- Design intent -

For seismic design, beam yielding defines demands
Joint demands

(a) moments, shears, axial loads acting on joint

(b) internal stress resultants acting on joint

(c) joint shear

Joint shear strength

\[ V_u = \phi V_n = \phi \gamma \sqrt{f_c' b_j h} \]
\[ \phi = 0.85 \]

ACI 352
Joint Details - Interior

Plan View of Connection (Top beam bars)

Elevation (Section A-A)

\[ h_{col} \geq 20d_p \]

ACI 352

Joint Details - Corner

Plan View of Connection (Top beam bars)

Elevation (Section A-A)

\[ \geq l_{dh} \]

ACI 352
Code-conforming joints

Joint failures
## Joint Design-ACI Requirements

**Table 11 - General Requirements**

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.5.1.1</td>
<td></td>
<td>Forces in longitudinal beam reinforcement at the face of the joint shall be determined assuming that the stress in the flexural tensile reinforcement is equal to 1.25$f_c'$.</td>
</tr>
<tr>
<td>21.5.1.2</td>
<td></td>
<td>Strength of joint shall be governed by the appropriate strength reduction factors $j$ in 9.5.4.</td>
</tr>
</tbody>
</table>
| 21.5.1.3  |          | Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored.*Tension according to 21.5.4*  
|           |          | *In compression according to Chapter 12*                                                                                                                                                                   |
| 21.5.1.4  | 11       | Where longitudinal beam reinforcement extends through a beam-column joint, the column dimension parallel to the beam reinforcement shall not be less than  
|           |          | *20 x diameter of the largest longitudinal bar for normal weight concrete*                                                                                                                                |
|           |          | *26 x diameter of the largest longitudinal bar for lightweight concrete*                                                                                                                                   |

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**Figure 11 - General Requirements and Transverse Reinforcement Requirements for Joints not Confined by Structural Members**

- $D_b$: beam diameter
- $D_c$: column diameter
- $D_s$: stirrup diameter
- $f_p$: peak stress in a FRP composite
- $f_r$: stress in reinforcing bars

*For lightweight concrete, multiply $f_r$ by 1.25*
*For epoxy-coated bars, multiply $f_r$ by 1.2*
Joint Design-ACI Requirements

Table 12 - Transverse Reinforcement Requirements

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
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</thead>
<tbody>
<tr>
<td>21.5.2.1</td>
<td>11</td>
</tr>
<tr>
<td>21.5.2.2</td>
<td>12</td>
</tr>
<tr>
<td>21.5.2.3</td>
<td>13</td>
</tr>
</tbody>
</table>

Transverse hoop reinforcement in 21.4.4 shall be provided within a joint, unless structural members confine the joint as specified in 21.5.2.2.

Where members frame into all four sides of a joint and each member width is at least \( \frac{2}{3} \) the column width, the transverse reinforcement within the depth of the shallowest member may be reduced to \( \frac{1}{2} \) of the amount required by 21.4.4.1. The spacing of the transverse reinforcement required in 21.4.9.2 shall not exceed 6 in. at these locations.

Transverse reinforcement per 21.4.4 shall be provided through the joint to confine longitudinal beam reinforcement outside the column core if a beam framing into the joint does not provide such confinement.

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Joint Design-ACI Requirements

![Diagram](image)

Figure 12 - Transverse Reinforcement Requirements for Joints Confined by Structural Members
Joint Design—ACI Requirements

Table 13 - Shear Strength Requirements

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.5.3.1</td>
<td>—</td>
</tr>
<tr>
<td>21.5.3.2</td>
<td>—</td>
</tr>
</tbody>
</table>

For normal weight concrete, $V$, of the joint shall not be taken as greater than:

- For joints confined on all four faces \( 20 \sqrt[4]{E_A} \)
- For joints confined on three faces or on two opposite faces \( 16 \sqrt[4]{E_A} \)
- For other joints \( 12 \sqrt[4]{E_A} \)

where $A_e$ = effective cross-sectional area within a joint computed from joint depth times effective joint width. The overall depth shall be the overall depth of the column. Effective joint width shall be the overall width of the column, except where a beam frames into a wider column, the effective width of the joint shall not exceed the smaller of 1. Beam width plus the joint depth.
2. Twice the smaller perpendicular distance from the longitudinal axes of the beam to the column side.

A joint is considered to be confined if the confining members frame into all faces of the joint. A member is considered to provide confinement at the joint if the framing member covers at least 75% of the joint face.

For lightweight aggregate concrete, the nominal shear strength of the joint shall not exceed 75% of the limits given in 21.5.3.1.
Joint Design-ACI Requirements

Table 1.6 - Development Length of Bars to Device

<table>
<thead>
<tr>
<th>Sect. No.</th>
<th>Fig. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 21.5.4.1</td>
<td>11</td>
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</tbody>
</table>

For normal weight concrete, the development length \( L_d \) for a bar with a standard 90-degree hook shall not be less than the largest of:
- \( 8 \times \text{diameter of the bar} \)
- 6 in.
- \( f_y / (f_{c,t, e}) \)

For bar sizes No. 3 through No. 11, the 90-degree hook shall be located within the confined core of a column or boundary element.

For lightweight aggregate concrete, the development length \( L_d \) for a bar with a standard 90-degree hook shall not be less than the largest of:
- \( 10 \times \text{diameter of the bar} \)
- 7.5 in.
- \( 1.35 f_y / (f_{c,t, e}) \)

For bar sizes No. 3 through No. 11, the 90-degree hook shall be located within the confined core of a column or boundary element.

For bar sizes No. 3 through No. 11, the development length \( L_d \) for a straight bar shall not be less than:
- \( 2.5 \times \text{diameter of the bar} \) if the concrete cast in one lift beneath the bar is 12 in.
- \( 3.5 \times \text{diameter of the bar} \) if the depth of the concrete cast in one lift beneath the bar is 12 in.

Straight bars terminated at a joint shall pass through the confined core of a column or boundary element. Any portion of \( L_d \) not within the confined core shall be increased by a factor of 1.5.

For epoxy-coated reinforcement, the development lengths in 21.5.4.1-21.5.4.3 shall be multiplied by:
- 1.5 for straight bars with cover less than 3x, or clear spacing less than Rd;
- 1.2 for all other straight bars;
- 1.0 for bars terminating in a standard hook.

Column Transverse Reinf. Design

\[
A_{sh} = 0.3 \left( sb_c \frac{f'_c}{f_y} \right) \left( \frac{A_g}{A_{ch}} \right)^{-1} - 1
\]

and

\[
A_{sh} = 0.09sb_c \frac{f'_c}{f_y}
\]

- \( A_g \) = gross area of column
- \( A_{ch} \) = area confined within the hoops
- \( b_c \) = transverse dimension of column core measured center to center of outer legs

Second equation typically governs for larger columns.
Column Transverse Reinf. Design

Maximum spacing is smallest of:

- One quarter of minimum member dimension
- Six times the diameter of the longitudinal bars
- $s_o$ calculated as follows:

$$s_o = 4 + \frac{14 - h_x}{3}$$

$h_x$ = maximum horizontal center to center spacing of cross-ties or hoop legs on all faces of the column, not allowed to be greater than 14 in.

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Column Transverse Reinf. Design

For max $s = 4$ in.

$$A_{shI} = 0.3 \left( \frac{sh_c f'c}{fy_t} \right) \left( \frac{A_g}{A_{ch}} \right) - 1 = 0.3 \left( \frac{4 \cdot 26.5}{60} \frac{900}{702} - 1 \right)$$

$A_{shI} = 0.60$ in$^2$

and

$$A_{shII} = 0.09sb_c \frac{f'c}{fy_t} = 0.09 \cdot 4 \cdot 26.5 \cdot \frac{4}{60} = 0.64$ in$^2$

Use 4 legs of #4 bar – $A_{sh} = 0.80$ in$^2$
Column Transverse Reinf. Design

Determine Seismic Shear

\[ V_{\text{seismic}} = \frac{8230 \cdot 2}{(12.5 \cdot 12) - 32} = 139 \text{ kips} \]

Note: \( V_{\text{seismic}} \approx 100\% V_u \)

\[ V_c = 0, \text{ if } P_{\text{min}} < \frac{f_c' A_y}{20} = 180 \text{ kips} \]

For 30 in. square column
\[ P_{\text{min}} = 266 \text{ kips} \text{ OK} \]
Column Transverse Reinf. Design

\[ \phi V_c = \phi 2^\lambda \sqrt{f'_c bd} = 0.75 \cdot 2 \cdot 0.85 \sqrt{4000} \cdot 30 \cdot 27.5 = 66.5 \text{ kips} \]

\[ \phi V_{s, required} = 139 - 66.5 = 72.5 \text{ kips} \]

\[ \phi V_{s, provided} = \phi \frac{A_y f_y d}{s} = 0.75 \frac{4 \cdot 0.2 \cdot 60 \cdot 29.6}{4} = 266.4 \text{ kips} \]

Hoops

4 legs #4

s = 4''

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Column Transverse Reinf. Design

Column Reinforcement

#4 hoops:

12 #8 bars